Optimization of Tunable Oscillators with AM to FM Conversion for Near-Carrier Phase Noise

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Abstract

Nonlinearity in the resonating tank of an oscillator results in conversion of amplitude fluctuations into frequency fluctuations. The effect of this AM to FM (AM2FM) transformation on phase noise is especially important in high-Q oscillators with a wide tuning range.

We develop quantitative time-variant model of near-carrier phase noise incorporating AM2FM conversion, which is both mathematically rigorous and intuitive, and propose new techniques to reduce AM2FM-induced noise in tunable oscillators.

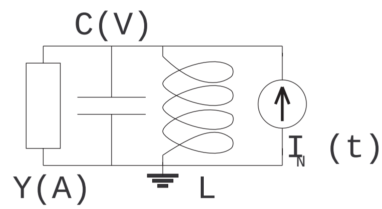

Figure 1: The two-port model of an oscillator.

1 Introduction

To quantitatively understand phase noise in autonomous systems, such as free-running oscillators, we need a time-variant theory [1]. For oscillators where the resonant circuits have high quality factor, such a model was described in [2]. In this paper, we present a simpler theory applicable to calculations of near-carrier phase noise in the vast majority of practical high-Q oscillators with a single resonator, including those with strong conversion of amplitude to frequency fluctuations (AM2FM), such as many wide-band tunable oscillators.

This paper also helps to develop intiutive understanding of its origins and discusses new techniques to reduce it. In section 2, the general formalism is briefly described. In section 3, we illustrate its applications to oscillators with fixed coefficient of AM2FM conversion. Section 4 is about ways to calculate and reduce AM2FM conversion.

2 The general formalism

In our description, the near-carrier frequency region corresponds to frequency offsets much smaller than the slowest relaxation rate in the oscillator, e.g., amplitude relaxation rate. At these offsets, the dominant contribution to phase noise results from slow random frequency modulation of the oscillator by internal and external noise sources [4]. Consider the oscillator representation shown in Figure 1. The resonant tank capacitance is assumed to be weakly nonlinear, i.e. for given tuning voltage, the relative variation in capacitance due to AC voltage across the tank is small. The combined effect of active device and parasitic tank conductance on the tank is modelled by nonlinear amplitude-dependent admittance $Y(A) = G(A) + jB(A)$. We assume that frequency dependence of $Y(A)$ is negligible in comparison with that of the admittance of the LC parallel circuit at the frequency of oscillations. We also omit $B(A)$ term in $Y(A)$, since in many cases it can be effectively included in $C(V)$.

The noise sources are referred to the tank and represented by the noise current $I_N(t)$. For near-carrier phase noise in an oscillator with $Q \gg 1$, it suffices to consider only near-carrier frequency components in $I_N(t)$, and I_N can be represented in terms of the in-phase $I_{nI}(t)$ and quadrature $I_{nQ}(t)$ constituents:

$$i_n(t) = Re(I_n(t)e^{j\phi(t)}) \ , \quad I_n = I_{nI}(t) + jI_{nQ} \ , \qquad (1)$$

where $\phi(t)$ is the phase of the voltage across the tank $V(t) = A\cos(\phi(t))$. If $I_{nI}(t)$ and $I_{nQ}(t)$ were constant, they would result in a constant shifts $\delta\Omega$ and δA in the amplitude and frequency of oscillations.

Without AM2FM conversion, i.e. at constant $C(V)$, $\delta\Omega$ and δA mostly depend on different noise components: $I_{nQ}(t)$ and $I_{nI}(t)$, respectively:

$$\delta A = \frac{I_{nI}}{AdG(A)/dA} \quad \delta\Omega = \frac{I_{nQ}}{2CA} \,. \tag{2}$$

An intuitive way to derive Eq. (2) is to compare unperturbed $V(t) = A_0\cos(\Omega_0 t)$, and perturbed $V(t) = A\cos(\Omega t)$ oscillations, where $\Omega_0^2 = 1/(LC)$, and A_0 satisfies $G(A_0) = 0$. The equation for perturbed oscillators

$$C\frac{dV}{dt} + G(A)V + \frac{\int V dt}{L} = I_{nI}\cos(\Omega t) - I_{nQ}\sin(\Omega t)$$

can be rewritten as

$$\left[C - \frac{I_{nQ}}{A\Omega}\right]\frac{dV}{dt} + \left[G(A) - \frac{I_{nI}}{A}\right]V + \frac{\int V dt}{L} = 0 \,.$$

Therefore, effect of quadrature noise current I_{nQ} is equivalent to changing the tank capacitance by $\delta C = -I_{nQ}/(A\Omega)$, which yields frequency shift given by Eq. (2). The amplitude shift is found from $G(A) - I_{nI}/A = 0$, which gives δA from Eq. (2)

In the presence of AM to FM conversion, instantaneous frequency of fluctuations Ω directly depends on amplitude shift δA. To intuitively understand this dependence, consider oscillations in an isolated lossless L$C(V)$ tank. The frequency of oscillations $\omega_T(A)$ depends on amplitude A because of capacitative nonlinearity. This relationship also holds for high-Q oscillator:

$$\delta\Omega = \frac{I_{nQ}}{2CA} + \delta A\frac{d\omega_T(A)}{dA} \,, \tag{3}$$

where δA is still given by Eq. (2), and $\omega_T(A)$ is the amplitude-dependent resonant frequency of the LC tank in the absense of $Y(A)$ and I_N, i.e. in purely conservative regime. Random processes $I_{nI}(t)$ and $I_{nQ}(t)$ result in random frequency modulation $\delta\Omega(t)$, which for the near-carrier spectrum is given by the quasistationary approximation, i.e. by substituting time-dependent $I_{nI}(t)$ and $I_{nQ}(t)$ into Eqs. (3).

For white noise in $I_n(t)$, phase noise can be characterized by phase diffusion coefficient D defined by

$$E[\Delta\phi]^2 = 2Dt \,,$$

where the l.h.s. is the average square of random phase walk during time t. If $I_{nQ}(t)$ and $I_{nI}(t)$ are independent, they give independent contributions to phase fluctuations, referred to as the direct and AM2FM phase noise, respectively, in this paper. In particular, D is then the sum of the direct and AM2FM phase diffusion coefficients:

$$D = D_{direct} + D_{AM2FM}$$

given by

$$D_{direct} = \frac{K_{nQ}}{16\pi C^2 A^2} \quad D_{AM2FM} = \frac{K_{nI}}{4\pi}\left[\frac{d\omega_T/dA}{dG/dA}\right]^2 \,. \tag{4}$$

3 Applications at a given amplitude-to-phase conversion

The ratio (D_{AM2FM}/D_{direct}) is proportinal to the squared quality factor Q of the tank. In time-domain picture, this quadratic dependence occurs because the amplitude relaxation time is proportional to Q. As a result, a given initial amplitude fluctuation affects the frequency during time $\propto Q$ and results in phase shift $\Delta\phi \propto Q$.

For frequency-domain derivation, consider oscillator with smooth nonlinearity

$$G(A) = \zeta(A^2 - 1) \,.$$

The unperturbed amplitude of oscillations is one, since $G(A = 1) = 0$. Let us assume that tank parameters are chosen for unit resonant frequency $\omega_T = 1$. The frequency shift due to in-phase noise is found from Eqs. (2, 3):

$$\delta\Omega = \frac{I_{nQ}}{2C} + \frac{I_{nI}d\omega_T(A)}{2\zeta dA} \,. \tag{5}$$

If C changes at constant ω_T, the tank quality factor is $Q \propto C$, which gives $D_{direct} \propto 1/Q^2$, while D_{AM2FM} is independent of Q. Therefore, the importance of AM2FM phase noise grows, as the quality factor of the resonator increases.

Another insight is obtained by noticing that $D_{AM2FM} \propto 1/\zeta^2$. Factor ζ characterizes the degree of oscillator nonlinearity under operating conditions. In time domain, larger gain nonlnearity ζ give faster amplitude relaxation and therefore smaller AM2FM phase noise (at constant Q).

As an example, consider a common-gate Colpitts oscillator with a long-channel square-law FET in Figure 2. If we consider phase noise due to thermal noise in the tank conductance G_T only, the one-sided power spectral density K_n of noise current I_n does not depend on conduction angle. It is given by Johnson-Nyquist formula and

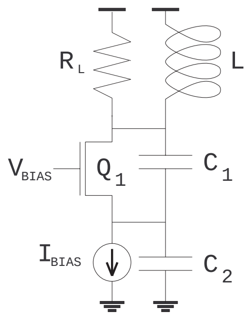

Figure 2: Common-gate Colpitts oscillator.

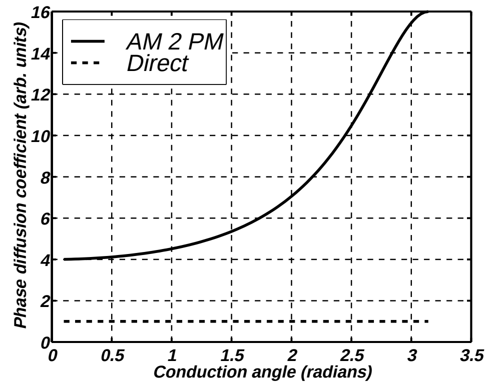

Figure 3: Phase diffusion coefficients due to tank-generated thermal noise

is splitted equally between in-phase K_{nI} and quadrature K_{nQ} components:

$$K_n = 4k_B T G_T = 2K_{nI} = 2K_{nQ}$$

which results in phase diffusion coefficients given by Eq. (4). Suppose that conduction angle θ decreases, while the amplitude of oscillations is constant. (This regime can be achieved by changing FET width and biasing current). As θ decreases, dG/dA and amplitude relaxation rate increase, which gives smaller D_{AM2FM}. The dependence of D_{AM2FM} on θ is shown in Figure 3 for long-channel FET with square-law IV characteristics. The tank has quality factor $Q = 50$ and AM2FM conversion coefficient $d\log\omega_T/d\log A = 0.02$. D_{direct} does not change with θ and is only given for comparison. In bipolar Colpitts oscillators, the dependence between tank-induced phase diffusion and conduction angle is similar. Finite resistance of the current source would slow down amplitude relaxation and therefore increase D_{AM2FM} [3].

Phase diffusion due to active device noise can also be calculated. The noise of active devices is usually cyclostationary rather than stationary [1], since it depends on periodically changing device state. For example, FET drain current noise i_{dn} occurs only when the device is not cut off. The analysis is further complicated, by the dependence of the amplitude of oscillations on low-frequency drain current noise [3]. In Figure 4, we plot phase diffusion coefficients *versus* conduction angle at constant oscillation amplitude, assuming that θ is changed by adjusting device width and bias current. Long-channel noise model is used [1]. Although the share of quadrature noise K_{nQ} in the total noise PSD K_n increases with the conduction angle [1], D_{direct} remains constant, because the total noise spectral density increases due to decrease in device width. Quadratic dependence of $D_{AM2FM}(\theta)$ at small θ occurs only for stiff current source.

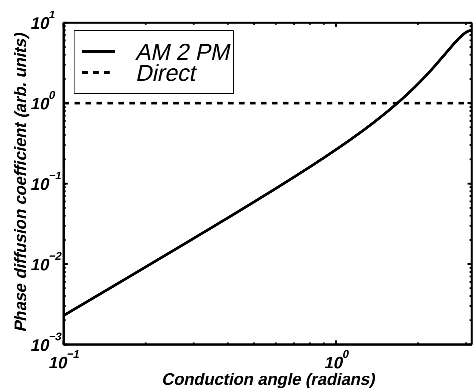

Figure 4: Phase diffusion coefficients due to long-channel FET noise

In bipolar Colpitts oscillators with stiff current source, dependence $D_{AM2FM} \propto \theta^4$ holds down to and saturates at $\theta \sim 1/\beta^{1/4}$ at constant current amplitude, where β is the low-frequency current gain, and the current amplitude of oscillations is assumed to be constant. At smaller θ, $D_{AM2PM}(\theta)$ is constant. Therefore, good *low-frequency* current gain can be important for good AM2PM noise performance of a bipolar Colpitts oscillator.

The decrease in AM2FM noise at small θ is essentially due to the constraint on current amplitude $A_I \approx 2I_{BIAS}$, which becomes increasingly rigid as θ approaches zero [3]. If the current noise has a moderate output impedance or is noisy, I_{BIAS} is not well controlled, and AM2FM noise can actually increase at small θ. A related phenomenon is reducing the direct phase noise by suppressing RF noise from the current source, e.g., by adding series inductor.

4 How to calculate and reduce AM2FM conversion

AM2FM phase noise can be reduced by adjusting nonlinearity in the tank capacitance to decrease the amplitude dependence of the resonant frequency ω_T of the isolated tank. In particular, for an LC tank with nonlinear capacitance $C(V)$, the dependence of the resonant frequency on voltage amplitude A is given by

$$\frac{d\omega_T(A^2)}{\omega_T d(A^2)} = \left[\frac{C'(V)^2}{12C^2} - \frac{C''(V)}{16C}\right] + O(A^2)\ . \quad (6)$$

In particular, cancelling the linear term in $C(V)$ does not necessarily reduce AM2FM noise proportional to $|d\omega_T/d(A^2)|^2$ and can, in fact increase it.

In practical oscillators, the tuning capacitor is usually connected to the tank through coupling capacitor C_C and is DC coupled with control voltage source through RF choke.In this case, $d\omega_T/d(A^2)$ is given by

$$\frac{d\omega_T(A^2)}{\omega_T d(A^2)} = \left\{\frac{C'(V)^2}{16C^2}\left[\frac{\eta}{3}+1\right] - \frac{C''}{16C}\right\}\zeta^2\eta^4\ , \quad (7)$$

where $\zeta = (C_C\|C)/(C_C\|C + (C_{\|})$, $\eta = C_C/(C_C + C)$. As C_C is reduced, AM2FM conversion rapidly decreases (approximately as η^4) due to decrease in both sensitivity of the total tank capacitance to $C(V)$ and the RF voltage amplitude across $C(V)$.

By properly choosing the nonlinear and coupling capacitors, as well as the range of control voltages for $C(V)$, $|d\omega_T/dA^2|$ can be significantly reduced, thereby decreasing AM2FM phase noise. As a theoretical example, the amplitude dependence of ω_T from Eq. (6) is canceled in the range of tuning voltages, where $C(V)$ is given by

$$C(V) = \frac{C_0}{(1 - V/V_B)^3}\ , \quad (8)$$

where C_0 and V_B are constants. Such a dependence is hardly possible to realize even for varactors with hyperabrupt junctions. However, for significant reduction of AM2FM phase noise it suffices to cancel $|d\omega_T/dA^2|$ only in the middle of the tuning range, a condition much easier to implement.

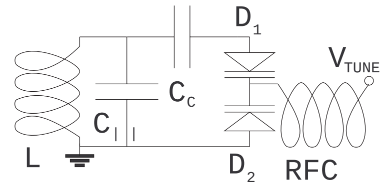

Figure 5: Antiseries varactor combination in the tank.

It is even possible to *completely* remove the dependence of the tank capacitance on the tank voltage, while maintaning tunability, i.e. the dependence of the tank capacitance on the tuning voltage V_{tune}. This surprising result, which was also independently obtained by Michael O'Neal, holds for the well-known antiseries connection (Figure 5) of two identical varactors, but with CV characteristics precisely given by $C_{var}(V) = C_0/\sqrt{1 - V/V_B}$, i.e. ideal CV for constant doping profile. Of course, the RF voltage across the varactors should always remain less than twice the build-in potential.

5 Conclusions

To summarize, we described an intuitive way to calculate phase noise in the presense of AM to FM conversion, discussed how phase noise depends on circuit parameters, and presented several new techniques to reduce near-carrier phase noise. More details will be given in [3]. The author is grateful to Michael O'Neal for helpful comments on the paper and discussions of phase noise.

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